Raganomicon

A Quantum Theory of Musical Scales

When you ferret out something for yourself, piecing the clues together unaided, it remains for the rest of your life in some way truer than facts you are merely taught, and freer from onslaughts of doubt.

Colin Fletcher
The Man Who Walked Through Time

Raganomicon

A Quantum Theory of Musical Scales

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Table of Contents

Preface

I want to make a reclaimer to modify the standard copyright disclaimer.
The charts and tables are designed to be copied, otherwise:
Why would I call one of them a cheat sheet?
It would be better for understanding, of course, to make hand written copies.
The last page is blank music paper; copying it can save
a lot of money for a starving student.
(Have you seen *the price of music paper recently?)*

Introduction

Do you worthy gentlemen now think that it's a small labor to inflate a dog?

Miguel de Cervantes Saavedra
Don Quixote, Part Two

THE VARIOUS STYLES OF MUSIC form various cults of fanatics. Jazz fans think that classical is reactionary and everything else is barbaric. Classical fans think that all pop music is banal and jazz is just another pop music. Rap fans think that nobody else understands the urban black experience. Metal fans think that nobody else understands the urban white experience. Narco fans think that nobody else understands the urban latino experience. Country fans think that nobody else understands the rural white experience. Ad nauseum. Just look at the list of styles that drops down on nearly any music (or social) website.

Question: What style of music do you prefer?

Answer: What style of music did you listen to when you were young?

My answer: A little bit of everything. In the 50s and 60s everything was all mixed up on the radio. Rock, country, jazz, blues, folk, gospel, etc. could all be heard on the same stations. Also, I began piano at age five and continued for seven years, before I began playing in bands, including marching band. I continued to study jazz theory and later also Indian theory. Later there was also church organ and collegiate jazz piano.

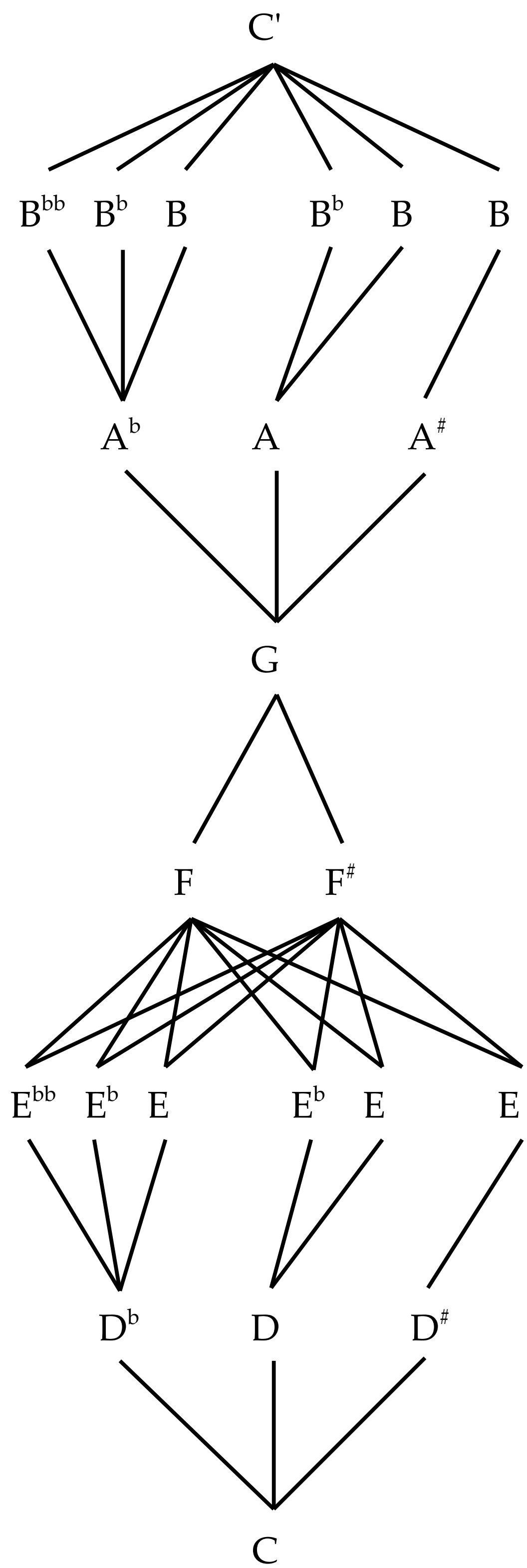
I include this nutshell description of my musical history in order to explain that I simply cannot listen to any style of music without analyzing it, nor can I attend to anything else but the music, no matter the style or quality. But the main point about popular music, no matter the style, is that these types of music are based on two things, dance and lyric song. Occasionally "instrumentals" appear on the pop charts, which replace lyrics with melody, but these works are almost always very danceable.

During the same period of time, I was also studying the modes within the groups, and learning their practical use for musical composition and improvisation. There is a wealth of material for the creation of new melodies (tunes) and harmonic cadences (chord progressions).

Music theory, throughout most of history, has come after the fact of musical practice, being mainly concerned with the structure and notation of music as it had already been written and/or improvised. Many of the greatest composers have also been great improvisers.

This booklet will describe a mental journey from confusion to precision, from an apparently random collection of musical scales to a perceptibly orderly classification of musical modes into a periodic table, plus the entire *Melakarta* System in Western notation. It also includes practical examples for composers and improvising musicians. Non-reading musicians can also use the intervals to explore these scales, and perhaps learn staff notation. Once you actually work with the scales as groups, a whole new world of music will open up for you. The direction and resolution of melodic movement will make sense in a new way, including, somewhat paradoxically, chromatic movement.

The *Melas* as Paths



Chapter 1: A Brief History of Exotic Scales

> Very well. He would indeed stay home with his strange Schoenbergian music which few people understood and even fewer enjoyed. He would think about hexachordal inversional combinatoriality and multidimensional set presentations, he would brood on the properties of the referential set, and let the rotting world go hang.
>
> Salman Rushdie
> *Two Years Eight Months and Twenty-eight Nights*

THE LURE OF THE EXOTIC has always been a powerful force, especially for the adolescent mind, dissatisfied with the present environment and seeking something different. For many of my generation, it was the sitar experiments by the Beatles and Stones. These led me to the music of Ravi Shankar, and later to other sitarists, such as Vilayat Khan. I was able to find Pundit Shankar's book and made a copy of the method section. I got a cheap sitar and wore it out.

Now Northern India and Southern India have related, but different systems of music theory. Many Northern theorists agree that the Southern theory is the more comprehensive of the two. The Southern is possibly the parent of the Northern theory, being a more direct survival of the older Hindu theory, before the Muslim invasion of the North. (In fact, the sitar is a hybrid between the Persian *sehtar* and the Indian *vina*.) In the Northern theory, the *ragas*, or melody types, are descended from *thats*, which can be translated as scales, or better as modes. There are about a dozen of these *thats*. In the Southern theory, the *ragas* come from 72 distinct *melas*, or modes, which include the Northern *thats*. The entirety of the Southern system is only known to elite scholarly musicians. On the other hand, the popular *ragas* (implying also the parent *melas* or *thats*) are very similar between both North and South. As we will see, these also include the Western scales and modes.

For instance *Mela Dhirasankarabharana*, *Bilaval That*, and the Ionian Mode, which is also the Major Scale, are the same pattern of notes. Historically, Indian music can be said to have preserved the ancient monophony, the single melody supported by a drone. (This simplified viewpoint ignores the importance of rhythm, which would, of course, require another book.) Western music, on the other hand, can be said to have undergone an evolution from monophony, through polyphony, to harmony, and subsequently into modern experimental movements such as minimalism and serialism. This suggests the possibility of a post-modern universalism which incorporates all of the music theories of the world, and resolves apparent contradictions. Such a Grand Unified Theory of Music would require the inclusion of the technical (written) aspects as can be gleaned from the literature of ethnomusicology. For instance, Indian music theory has has an extensive and surprisingly systematic literature on the emotional effect of music; its fascinating content ranges through philosophy, poetry, dance, painting, sculpture, architecture, indeed, all of the arts. The *ragamala* paintings of the Moghul courts, are the most famous example. We must, alas, restrict our analysis to those aspects of melodic practice which can be represented mathematically and geometrically. Another Muslim invasion, the occupation of Spain, was responsible for the spread of Eastern music into Europe over an extended period. The guitar evolved from the *oud* (lute), and exotic scales entered the common practice from the Spanish Gypsy melodies. The tune “Greensleeves,” in the

Harmonic Minor, is perhaps the most memorable example of this ubiquitous influence.

In modern times the story of Claude Debussy's revelatory experience of Indonesian *gamelan* music has been told and retold. The *slendro* scale is very close to the Whole Tone Scale in Western music. These scales have been described as "static," which I take to mean that the strong movement of harmony and melody typical of the Major or Minor Scales of Western music is replaced by a more neutral tonality. The *gamelan* scales seem to me to have evolved to fit the harmonic characteristics of metallophones, as opposed to strings and pipes. The word *gamelan* applies not only to the musical tradition itself, but also to the actual sets of instruments, which are custom cast and precisely tuned and retuned by hammering and filing. No two sets are alike. In his survey *Music Cultures of the Pacific, the Near East, and Asia*, William P. Malm states that "The search for musical categorizations can begin by turning to literate societies where specific theoretical explanations are found for many tonal and compositional principles. These principles cluster around three of the four major written music-theory systems of the modern world: the Arab-Persian, Indian, and Chinese. The fourth, the Western, shares with the Near Eastern traditions certain historical roots in the Greco-Roman world." He then goes on to add, "A fifth large unit may be the knobbed-gong culture of Southeast Asia, since it represents distinctive musical styles and instruments..." This fifth musical culture being the *gamelan*.

I intend to show a classification of the *Melakarta* system into scale groups, which could also be called related keys. The scales are also "translated" into Western notation, both staff, and an extended Roman numeral "shorthand." I will also show that the Western scales are subsets of the Southern Indian system, and that the scales form themselves into periodic groups. After working out the relationships among the 72 *Melakartas,* I found that Southern Indian Theory already had a classification of the *Melakartas* into *Grabdeha* Groups. My initial disappointment at not being the original discoverer of these relationships was tempered by the fact that I had done this work independently, and that my groups aligned perfectly with the *Grabdeha* groups. All this was part of my original goal, or rather process, of learning the scales well enough to compose melodies. The idea of making the *Melakarta* system more available to other Western musicians evolved later.

A major part of my efforts in the study of music theory has been the translation of the *Melakarta* system of 72 *Melas* (modes or scales) into standard Western notation. Of course this is easy enough, all you have to do is write out all of the modes in staff notation. But then I noticed that some of the modes fell into groups, and puzzlingly, the members of these groups were widely separated on the Table of *Melakartas*. There was no obvious way to connect the members of these groups, although they were determined in an exact and orderly manner, their relationships among each other seemed to be randomly spaced. Was there any way to make sense of this apparent chaos? I began to think about various ways to classify the modes.

The path of innovation in the nontechnical areas of musical composition has followed a different course altogether. Twelve Tone and Serialist composition uses esoteric numerological procedures to write music for traditional instruments. Schoenberg's *Structural Functions of Harmony* is possibly the last word on classical and neoclassical forms of composition. And yet, his final chapter looks forward to a theory to explain Twelve Tone composition. We might want to set out to analyze the Twelve Tone system. There are 479,001,600 possible twelve tone rows. This is a small step compared to, say, the number of stars in the universe, but a giant leap to learn all of these "scales." Such an analysis would be beyond the scope of this booklet; it would require the identification and classification of all the 479 million tone rows. You would have to write an app for that.

Innovative advancement in the design and construction of instruments has also been both challenge and inspiration to musicians. Equal temperament for keyboard and fretted instruments is the classic example of this influence; but there are many more including mechanical keys for woodwinds, valves for brass, and pedals for the harp. Most, if not all, technical innovations lead to more precise control of each individual instrument, as well as the ability of ever larger ensembles to play together in tune. Of course, these are all historical, belonging to the period of Classical Common Practice.

In the Twentieth Century, technical innovation followed the course of the electronics industry. Music synthesizers have made it possible to simulate the sound of any instrument, as well as to create new sounds. Software is available for any style of composition and recording. The only limits are the cost of the equipment and the imagination of the artist.

It was bad enough having to learn 24 scales (Major and Minor in all 12 keys) as a young student, and later learning 72 scales (all 6 modes in all 12 keys). Then we have the 72 scale/mode system of *Melakartas*, which gives a total of 72 scales times 12 keys, 864 total. Instead of this top down approach we can use a bottom up method. What I hope to present here is a relatively understandable explanation of a "quantum" theory of musical scales, complete with its own "periodic" table. The 72 *Melakartas* can be regarded as analogous to the 92 Natural Chemical Elements. There are simple mathematical reasons that these systems form the particular shapes that they do; in chemistry it is the number of protons in the nucleus; in music it is the number and the order of the intervals. And using the same math, I also found another 48 scales which I have named the *Ekamelakartas*. I could also call this a "grand unified theory," but it is based on only two systems: South Indian and Western—it says nothing about the other major music theories. Or I could call it a "string theory," disregarding the pun, because the scales as regarded as intervals and/or positions are strings in the mathematical and computational definitions.

Of course, this is an analysis of the structural properties of the scales with regard to melody and harmony; it says nothing about time and rhythm. An analysis of rhythm would take a completely different approach, because musical rhythms are based on the internal cycles of the human body like brain waves, heart rates, breath rates, as well as kinesthetic motion. My theory might be related, by a long stretch, to Music Set Theory. It is closer to Diatonic Theory, which counts intervals in a similar way, as well as building groups of harmonic transformations.

Schoenberg's Regions

Major

$G^{\#}$												D^{b}
$g^{\#}$												d^{b}
	E		e		G		g		B^{b}		b^{b}	
$C^{\#}$												G^{b}
$c^{\#}$												gb
	A		a		C		c		E^{b}		e^{b}	
$F^{\#}$												C^{b}
$f^{\#}$												c^{b}
	D		d		F		f		A^{b}		a^{b}	
B												F^{b}
b												f^{b}

Minor

		G		e		E		$c^{\#}$		$C^{\#}$
c		C		a		A		$f^{\#}$		$F^{\#}$
f		F		d		D				

Chapter 2: A Brief History of Western Theory and Notation

> Our two main tonalities, major and minor derive historically from the church modes. The contents of the three major-like modes—Ionian, Lydian and Mixolydian—are concentrated in the one major tonality, and the contents of the three minor-like modes—Dorian, Phrygian and Aeolian—in the same manner, are concentrated in the minor.
>
> Arnold Schoenberg
> *Structural Functions of Harmony*

SINCE THE DAYS OF J. S. BACH, Western music has been defined by the Tempered Scale, and the teaching of theory defined by the keyboard. After many years of debate about the relative merits of the Pythagorean, Just, and Tempered systems of tuning; the Tempered system won out for keyboard instruments and large instrumental ensembles, but the Just system still turns up in smaller ensembles, particularly vocal and bowed strings. I personally think that the slight dissonance between a vocal group and a keyboard based ensemble creates an additional layer of complexity to the overall sound. In fact, it is this very phenomenon that accounts for much of the perceived differences among various pop music genres; the vocal styles make more of an impression on the general listener than the instrumental.

Bach wrote his *Well Tempered Clavier* as a musical proof of the Tempered system, twelve pieces in Major keys, and twelve in the related Minor keys. Here, we will only be concerned with the key of C; all of the scales based on C can be transposed into the other eleven.

Gentle reader, if you who are familiar with music theory, please bear with me; if you are unfamiliar with music theory, I will try to present the basics. There will be some simplification, but also some tightening and loosening of definitions. This will clarify some of the more confusing terminology. Remember that music theory, as it exists today, has evolved over centuries and contains many ambiguities and apparent contradictions. Here is the Middle C octave of a keyboard instrument, and the first example of an ambiguous notation:

Black Keys:		C#/Db		D#/Eb				F#/Gb		G#/Ab		A#/Bb			
White Keys:	C		D		E		F		G		A		B		C'
Interval:		Whole Step		Whole Step		Half Step		Whole Step		Whole Step		Whole Step		Half Step	
Number:	1		2		3		4		5		6		7		1

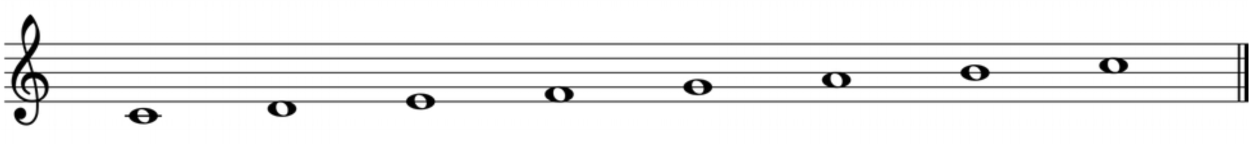
Here is the same Middle C octave on a fretted instrument with one string tuned to C:

Note:	C	C#/Db	D	D#/Eb	E	F	F#/Gb	G	G#/Ab	A	A#/Bb	B	C'
Fret:	0	1	2	3	4	5	6	7	8	9	10	11	12
Interval:			2		2	1		2		2		2	1
Number:	1		2		3	4		5		6		7	1

So we see that on a fretted instrument, one fret equals one Interval. (Of course, the actual frets are built with continually decreasing spacings. This is due to the physics of sound, and need not concern us here and now.)

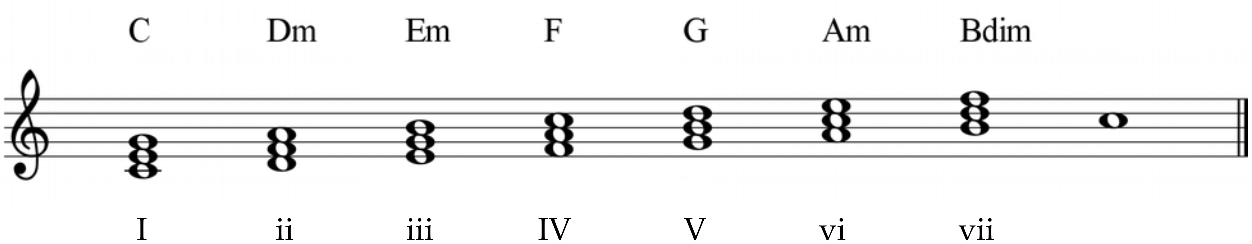
The Whole Step is two Intervals, and the Half Step is one! And, just to add to the confusion, Whole Step and Half Step are also known as Tone and Semitone, which mean exactly the same thing. So for our purposes, we will use the term Interval, and forget about the whole Whole Step-Half Step/Tone-Semitone "ball of confusion." So a Whole Step is two Intervals and a Half Step is one Interval. We already have four different notations, the letters A - G plus the # and b symbols, (starting with C, another contradiction), the fret numbers, the Intervals, and the Scale numbers. We will need two more notations, Staff and Roman numerals. I will use Arabic numerals for the intervals, and Roman numerals for the scales/modes/*melas*. I will use "(mod 12) + 1" for the chromatic scale, and "(mod 7) + 1" for the diatonic scale.

Staff notation is the most widely used. We will confine our discussions to the Treble Clef. The Staff refers to the lines, and the Clef refers to the symbol. The Treble Clef is also known as the G Clef, because the inside curl of the symbol curls around the second line from the bottom, which is the note G. Here is the Major Scale:



The deceptive perception of this system is that all of the steps appear to be equal, even though they are made up of varying intervals. Having mentioned this deception, it remains the best system for large ensembles; it keeps everybody on the same page.

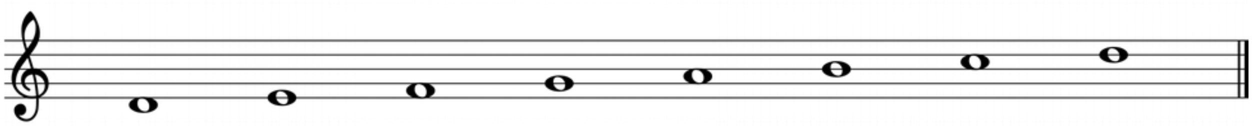
When two or more notes are played simultaneously, we call it a chord. Chords can be represented either by letters or by Roman numerals. Here are the common triad chords that go with the Major Scale; The letter names are shown above the staff, and the Roman numerals below:



The Roman numerals have two advantages over the letter names. The most important is that they do not depend on the key signature. That is, they remain the same for the key of C (shown), or the key of D, or any other of the twelve major keys. The other advantage is that the upper case denotes a major chord, and the lower case denotes a minor chord. (Some writers use all upper case.) The Roman numerals can also represent the Greek modes. The modes can also be confusing, because the names have changed throughout history. Nobody is really sure just what the original Greek modes were, then there were the Byzantine modes, and the Gregorian modes. We now use the Greek names for the modern modes, even though they are arbitrarily named. I prefer to use the Roman numerals. The Major Scale pictured above is now also known as the Ionian mode. Here are the modes formed by displacement:

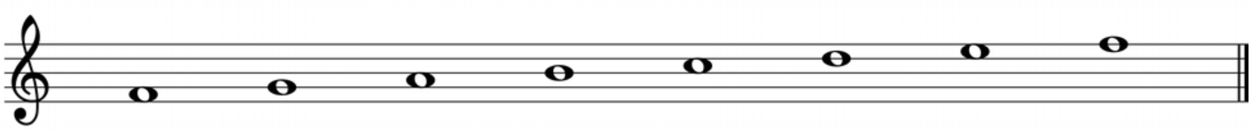
Ionian mode or I, the Major Scale

Dorian mode or ii

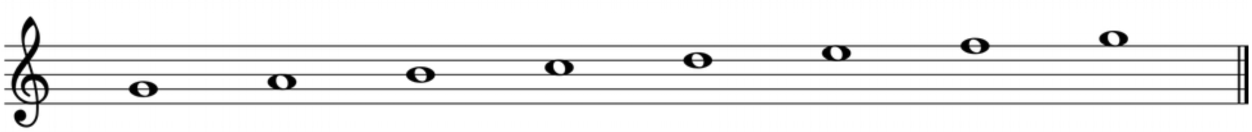


Phrygian mode or iii

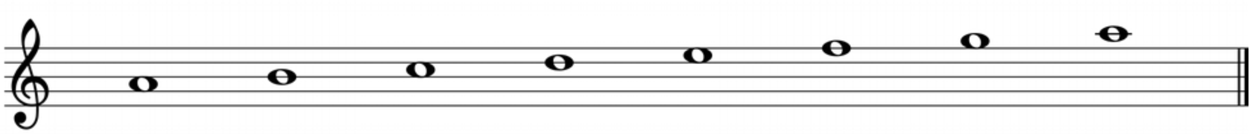
Lydian mode or IV



Myxolydian mode or V



Aeolian mode or vi, the Minor Scale

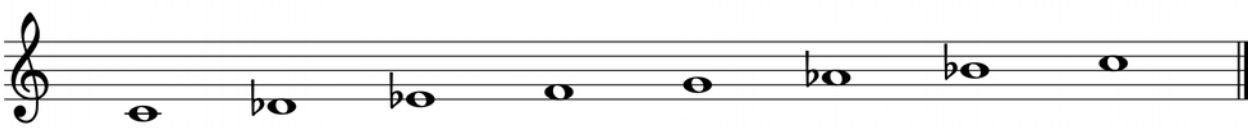


The Locrian Mode has been left out because it lacks a perfect fifth; the reason for this will be explained in the next chapter. The modes can also be represented by alteration (changing one or more notes to sharp or flat), which maintains the tonic tone. Here are the modes formed by alteration:

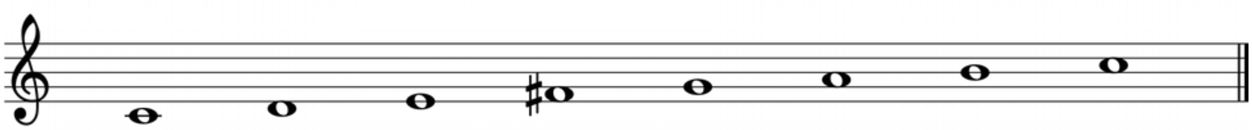
Ionian mode or I, Key of C Major

Dorian mode or ii, Key of B^{b} Major

Phrygian mode or iii, Key of A^{b} Major

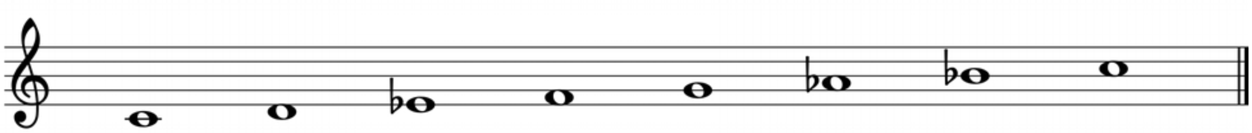


Lydian mode or IV, Key of G Major



Myxolydian mode or V, Key of F Major

Aeolian mode or vi, Key of C Minor (E^{b} Major)



The modes built by alteration can also be referenced back to Schoenberg's regions with regard to the key signatures. The central keys of e, G, a, C, d, and F could mean the six modes as just described, but more probably meant the vi of I in each of the keys G, C, and F. In his analysis of traditional harmony, Schoenberg introduces the principle of monotonality, which regards every change from the tonic to remain within the tonic, whether the harmony is closely or distantly related.

> Monotonality includes modulation—movement towards another *mode* and even establishment of that mode. But it considers these deviations as regions of the tonality, subordinate to the central power of a tonic. Thus comprehension of the harmonic unity within a piece is achieved.

The emphasis on *mode* means, to me, a *melodic* mode, which is understood to be supported by a *harmonic* structure. In classical practice, certain chord progressions were *verboten*, for instance a change from V to IV (or vice versa) was not allowed, there needed to be a ii interposed. In current pop music the V-IV progression is almost *de rigueur*. Today, most musicians consider a modulation to be a complete change of key, although possibly temporary with a return to the original tonic, or possibly followed by another modulation.

The history of Western music is usually described as the evolution from a monophonic melody line, to polyphonic melody lines, to harmonically constructed melody lines, to harmonically supported melody lines (these understood to remain in the established key), then to logical modulations, to freely invented modulations, and finally to abstract mathematical methods of composition. Examples of these styles, in order, are Gregorian chant, Palestrina, Bach, Mozart, Beethoven, Wagner, and Schoenberg. I have argued earlier that the technology of musical instruments has been a major driving force behind the developments of new styles of music. The sheer size increases of the ensembles proves this point, ranging as it does from a single cantor up to enormous orchestras and choirs.

Beethoven's Ninth has often been cited as the classic example of logical modulations, the harmonic (chord) progression goes through all 24 of the Major (I) and Related Minor (vi) Keys:

C a F d B^b g E^b c A^b f D^b b^b G^b e^b B a^b E c$^\#$ A f$^\#$ D b G e (C')

This recalls Bach's first use of the full set of Western keys, and in a sense, the culmination of the Western Classical tradition.

Wagner introduced a new method of composition, where the melody wandered in and out of the key, and the harmony followed along with the key changes. It was this Post-Romantic style that challenged Schoenberg's analytical skills and led to his conception of monotonality.

The *Melacarta* Table

		B^{bb} A^{b}	B^{b} A^{b}	B A^{b}	B^{b} A	B A	B $A^{\#}$
D^{b} E^{bb}	F	1	2	3	4	5	6
D^{b} E^{b}		7	8	9	10	11	12
D^{b} E		13	14	15	16	17	18
D E^{b}		19	20	21	22	23	24
D E		25	26	27	28	29	30
$D^{\#}$ E		31	32	33	34	35	36
D^{b} E^{bb}	$F^{\#}$	37	38	39	40	41	42
D^{b} E^{b}		43	44	45	46	47	48
D^{b} E		49	50	51	52	53	54
D E^{b}		55	56	57	58	59	60
D E		61	62	63	64	65	66
$D^{\#}$ E		67	68	69	70	71	72

Chapter 3: A Brief History of the Southern Indian *Melakarta* System

> [Music] has become the most theoretical and formally structured of our major art-forms. While the prospective painter or writer can begin ambitious creative work at once, the aspiring musician must be immersed more deeply in the rules and theory of music before any coherent beginning is possible.
>
> John D. Barrow
> *The Artful Universe Expanded*

SOME OF US, musicians or not, probably remember the solfege (do-re-mi) system from childhood. Both the North and the South Indian systems use a very similar syllabic notation:

C	D	E	F	G	A	B	C'
Do	Re	Mi	Fa	Sol	La	Ti	Do
Sa	*Ri*	*Ga*	*Ma*	*Pa*	*Da*	*Ni*	*Sa*
1	2	3	4	5	6	7	1

This basic scale, called *Bilaval* in the North, *Dhirasankarabharana* in the South, is the same as the Western Major Scale. The tunings may differ, they might be Just and/or Pythagorean, and the Tonic may be "between the cracks" of a piano, but the scale "remains the same."

There is also a notation for flat and sharp, *komal* and *tivra*, respectively. The *Melakarta* system is built by tetrachords, the lower of C to F, and the upper of G to C'. C and G are always required, these notes, which we call Tonic and Fifth, create the drone substrate of most Indian music. The alterations of the other notes are: D may become D^b or $D^\#$, E may become E^b or E^{bb}, F may become $F^\#$, A may become A^b or $A^\#$, and B may become B^b or B^{bb}. This works out to a block of 36 scales with F natural and another block of 36 with F sharp.

The scales (*Melas*) are numbered from 1 to 72. There is a system of naming the *Melas* in Sanskrit, in which language each letter of the alphabet has a numerical value. A South Indian musician who is educated in this system can determine the scale from the name. I pretty much slept through my Sanskrit classes, so I just go by the numbers.

The table on the facing page shows the organization of the *Melas* that I have just described. The upper block and the lower block are the same except for the F. The diagram is actually little more than a list of scales. In order to understand the different scales we must work with them individually and in groups. Get to know them, so to speak. We will also analyze the symmetry of the scales and place them in different sets and subsets based on their similarities.

I have presented the names of the *Melas* for those readers who may be interested in learning them. Walter Kaufmann's book, *Musical Notations of the Orient*, includes an excellent description of the Sanskrit letter codes used to name the *Melas*. There is a long history of the gradual development and acceptance of the *Melakarta* system by the pundits of South India. It has even been accepted among many musicians in North India, where it serves as a source for new *raga* compositions. As a logical, rational system, it only makes sense to introduce it to Western musicians. We will see that the Western system of scales is a small subset of the *Melakarta* system (roughly 12 out of the 72 or 1/6), as are most other systems of scales. For instance, the North Indian system of scales includes most of the Western scales, plus a few others, but all of these are within the *Melakarta* system, The Chinese, Japanese, Persian, and Arabic systems can all be described within the *Melakarta* system.

The *Melakarta* system can be visualized as a map of possible paths through the octave as the illustration facing Chapter 1. By following a path from bottom to top, or vice versa, we can build any one of the 72 *Melas*. This makes a very pretty picture, but it tells us nothing about the relationships between the individual scales. If we wish to investigate the relationships among the 72 *Melas*, we must use the numerical chart.

In the previous chapter on Western Theory, we saw that the modes formed from the Major Scale by displacement become their own subsystems (modes) within any particular key. In the next chapter we will see that this process can be applied to all of the *Melas*, with widely varying results. (It can also be applied to other scale systems.)

It should be borne in mind that classical Indian music, both Southern and Northern, are essentially monophonic; they employ drones of the tonic and fifth (more rarely tonic and fourth), as well as percussion tuned to the tonic. The vocalists and instrumentalists sing and play within the structure established by the tonic. Each note is heard in relation to the drone and the level of tension between the melody notes and the tonal environment.

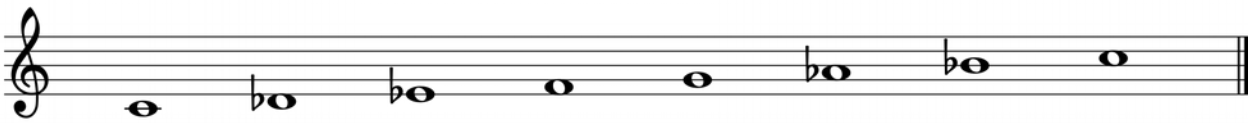
Of course this all tells very little about the actual performances. Indian music has prominent percussion; the drummer plays intricate patterns and provides not only accompaniment for the melodic performers, but also changes in tempo to delineate different sections of the music, and also drum solos which may depart from the main beat and work back into the flow of the song. Percussive virtuosity is appreciated as much as melodic virtuosity.

Because classical Indian education in music consists of direct instruction from master to apprentice, each student learns a regional and "school" style, each of which have their own interpretations of the *ragas*. The student must also learn the *talas* (rhythmic patterns).

The *melas* provide the scale structures from which the *ragas* are derived. The *ragas* are somewhere in between traditional melodies and melody types. A Western musician might think of them as something akin to folk tunes or hymn tunes, or any melody that has passed from copyright into the public domain. They are essentially a framework upon which the musician builds a structure. The similarity, and sometimes identity, of folk tunes and hymn tunes is demonstrated by my earlier example of "Greensleeves," which also became the hymn "What Child is This." Indian *ragas* go through a similar process of transformation.

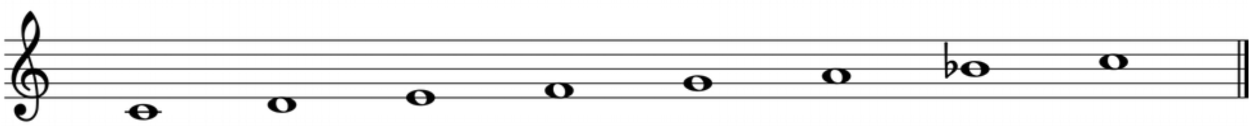
Universal Group

8, iii, *Mela Hanumatodi, Bhairavi That,* Phrygian Mode



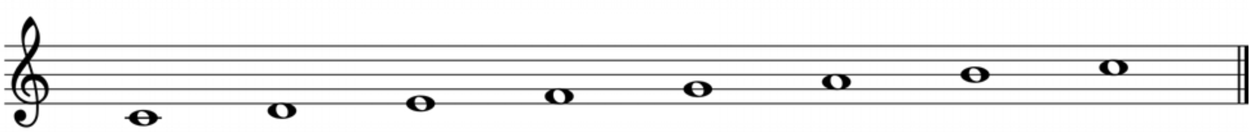
65, IV, *Mela Mechakalyani, Kalyan That,* Lydian Mode

28, V, *Mela Harikambhoji, Khamaj That,* Myxolydian Mode

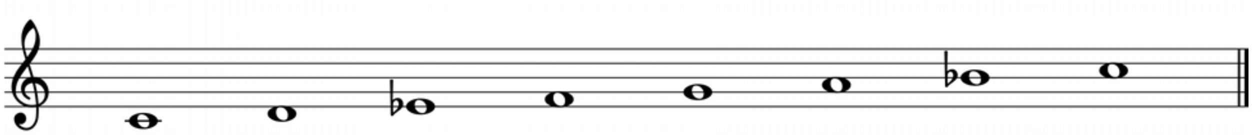


20, vi, *Mela Natabhairavi, Asavari That,* Aeolian Mode, Natural Minor

29, I, *Mela Dirasankarabharanam, Bilaval That,* Ionian Mode, Major Scale



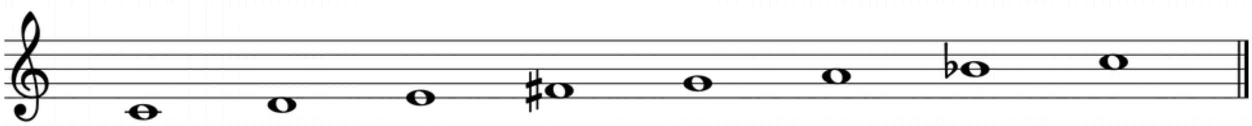
22, ii, *Mela Kharaharapriya, Kafi That,* Dorian Mode



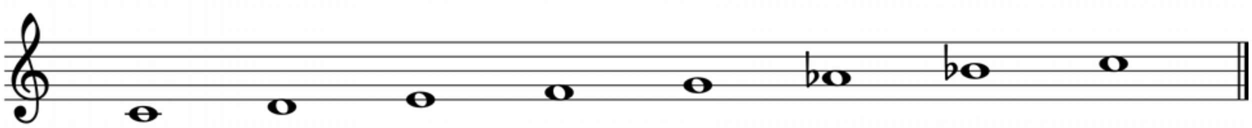
Overtone Group

10, iii$^{\#}$6, *Mela Natakapriya,* Jazz Minor Inverse

64, IV^{b}7, *Mela Vachaspati,* Overtone Scale, Lydian Dominant Mode



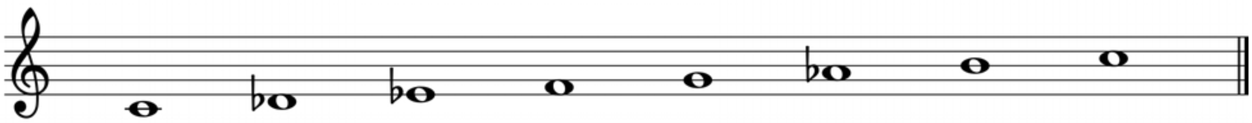
26, V^{b}6, *Mela Charukesi*



23, ii$^{\#}$7, *Mela Guarimanohari,* Ascending Melodic Minor, Jazz Minor

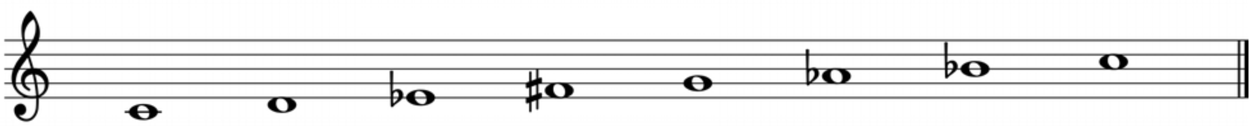
Neapolitan Minor Group

9, iii$^{\#}$7, *Mela Dhenuka,* Neapolitan Minor

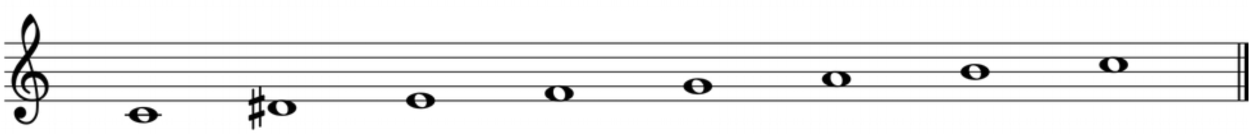


66, IV$^{\#}$6, *Mela Chitrambari*

56, vi$^{\#}$4, *Mela Sanmukhapriya*



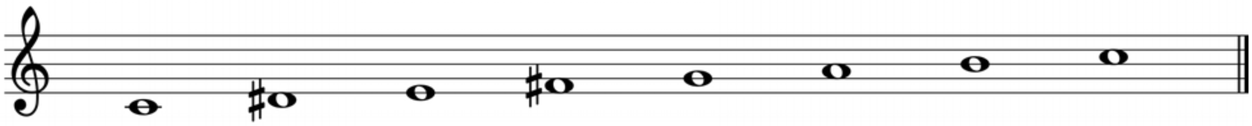
35, I$^{\#}$2, *Mela Sulini*



Harmonic Minor Group

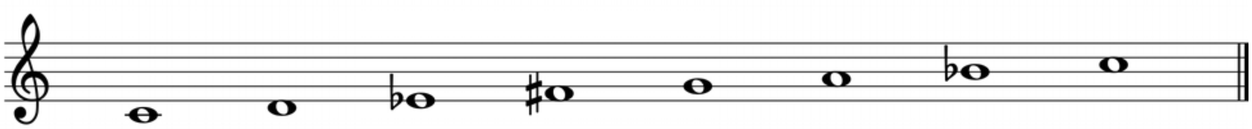
14, iii$^{\#}$3, *Mela Vakulabharanam,* Harmonic Major Inverse

71, IV$^{\#}$2, *Mela Kosalam*



21, vi$^{\#}$7, *Mela Kiravani, Pilu That,* Harmonic Minor, Spanish Gypsy

58, ii$^{\#}$4, *Mela Hemavati*

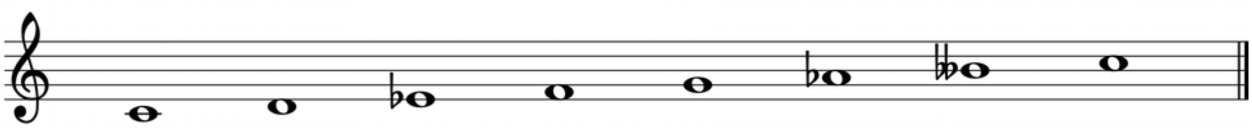


Marava Group

2, $\mathrm{iii}^{b}3$, *Mela Ratnangi*

53, $\mathrm{IV}^{b}2$, *Mela Gamanasrama, Marava That*

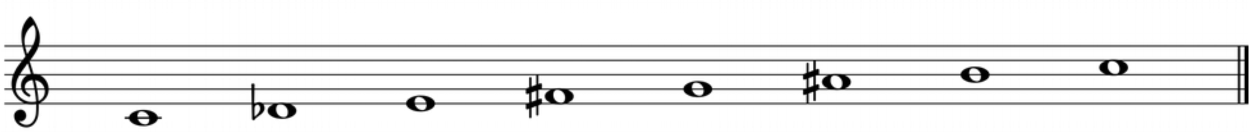
19, $\mathrm{vi}^{b}7$, *Mela Jhankaradhavani*



Visvambari Group

3, $\mathrm{iii}^{b}3^{\#}7$, *Mela Ganamurti*

54, $\mathrm{IV}^{b}2^{\#}6$, *Mela Visvambari*



55, $\mathrm{vi}^{\#}4^{b}7$, *Mela Syamalangi*

Bhairabuhar Group

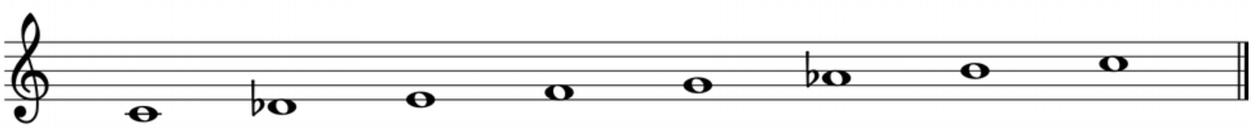
7, iii^{b}7, *Mela Senavati*

63, IV^{b}6, *Mela Latangi*

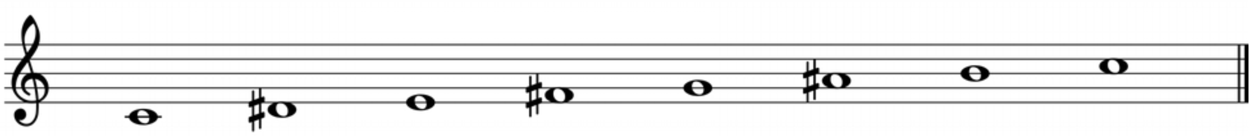
17, I^{b}2, *Mela Suryakantam, Bhairubahar That*

Bhairava Group

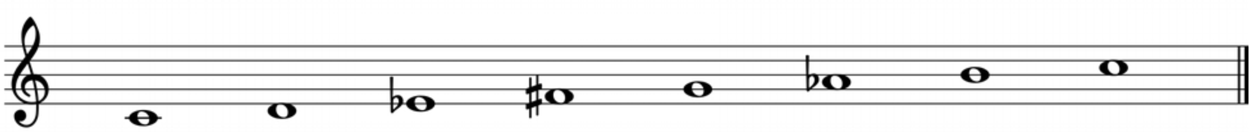
15, I^{b}2^{b}6, *Mela Mayamalavagaula,* Gypsy Major, Double Harmonic Major



72, IV$^{\#}$2$^{\#}$6, *Mela Rasikapriya*



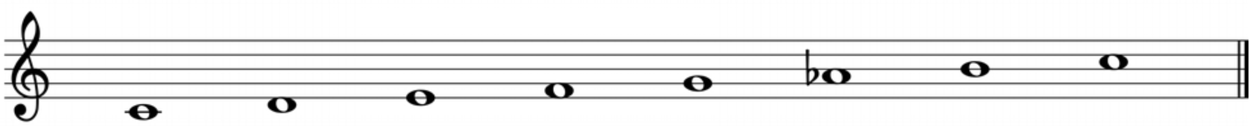
57, vi$^{\#}$4$^{\#}$7, *Mela Simhendramadhyama,* Gypsy Minor, Double Harmonic Minor



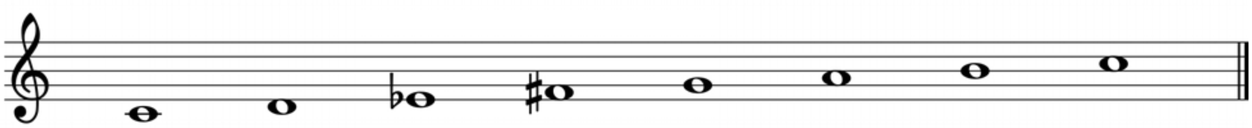
Harmonic Major Group

16, V^{b}2, *Mela Chakravakam,* Harmonic Minor Inverse

27, I^{b}6, *Mela Sarasangi,* Harmonic Major



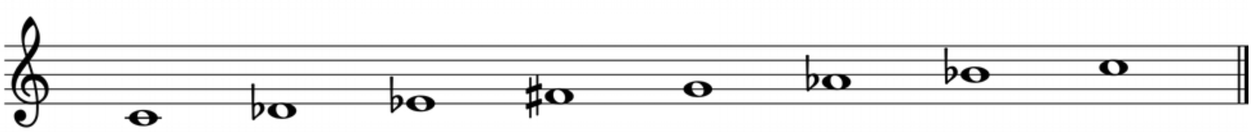
59, ii$^{\#}$4$^{\#}$7, *Mela Dharmavati,* Lydian Diminished



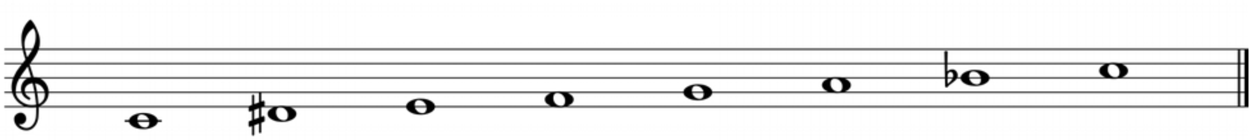
Vagadhisvani Group

30, I$^{\#}$6, *Mela Naganandini*

44, iii$^{\#}$4, *Mela Bhavapriya*



34, V$^{\#}$2, *Mela Vagadhisvari*



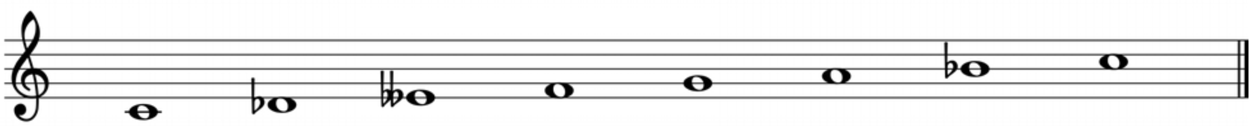
Shri Group

1, iii^{b}3^{b}7, *Mela Kanakangi,* Chromatic Dorian Mode

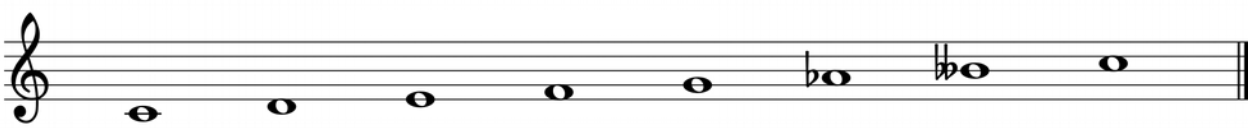
51, IV^{b}2^{b}6, *Mela Kamavardhani, Shri (Purvi)That,* Chromatic Hypolydian

Vanaspati Group

4, iii^{b}3$^{\#}$6, *Mela Vanaspati*

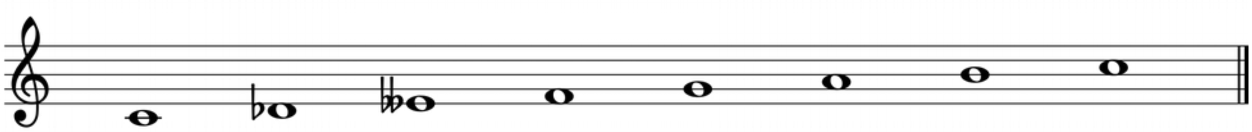


25, V^{b}6^{b}7, *Mela Maranjani*



Manavati Group

5, iii^{b}3$^{\#}$6$^{\#}$7, *Mela Manavati*

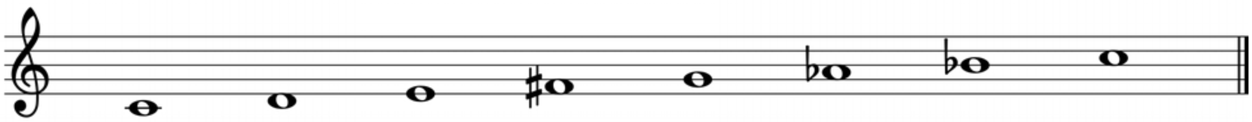


61, IV^{b}6^{bb}7, *Mela Kantamani*

Whole Tone Group

11, iii$^{\#}$6$^{\#}$7, *Mela Kokilapriya,* Whole Tone Minor

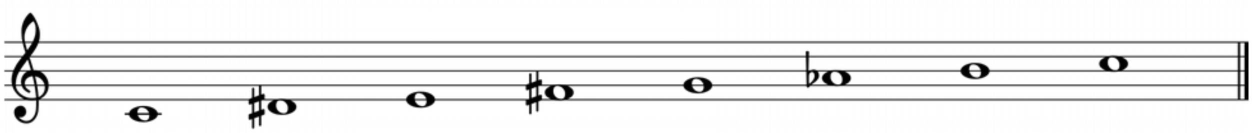
62, IV^{b}6^{b}7, *Mela Risabhapriya,* Whole Tone Major



Gayakapriya Group

13, iii$^{\#}$3^{b}7, *Mela Gayakapriya,* Gypsy Hexatonic

69, IV$^{\#}$2^{b}6, *Mela Dhatuvardhani*



Hatakambari Group

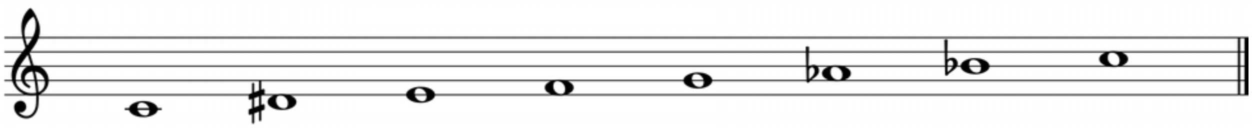
18, I^{b}2$^{\#}$6, *Mela Hatakambari*

43, iii$^{\#}$4^{b}7, *Mela Gavambhodi*

Varunapriya Group

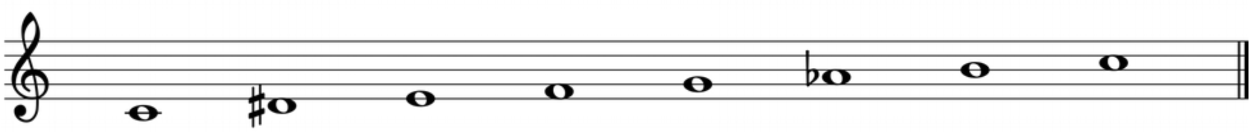
24, ii$^{\#}6^{\#}7$, *Mela Varunapriya*

32, V$^{\#}2^{b}6$, *Mela Ragavardhani*



Nitimati Group

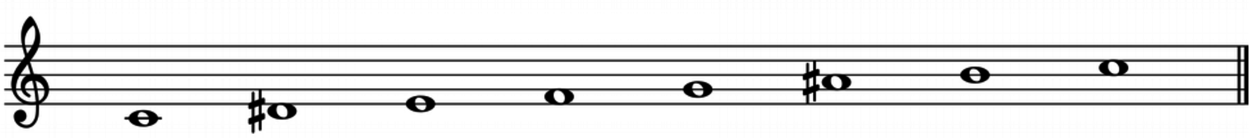
33, I$^{\#}2^{b}6$, *Mela Gangeyabhusani*



60, IV$^{b}3^{\#}6$, *Mela Nitimati*

Todi Group

36, I$^{\#}2^{\#}6$, *Mela Chalanata,* Chromatic Dorian Inverse

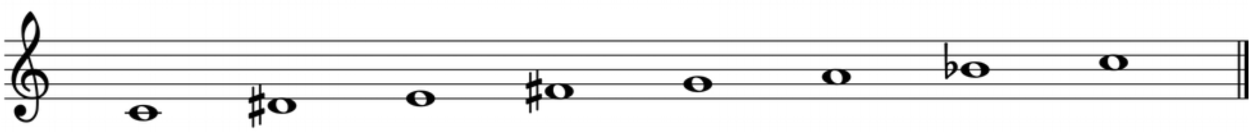


45, iii$^{\#}4^{\#}7$, *Mela Subhapantuvarali, Todi That,* Chromatic Lydian Inverse

Hungarian Major Group

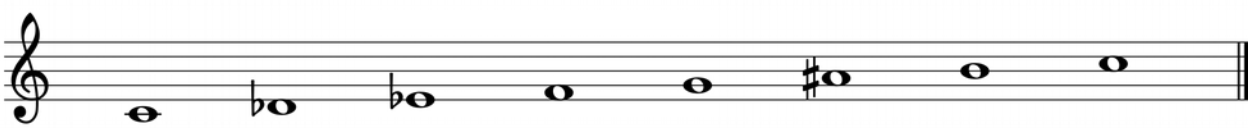
46, iii$^{\#}$4$^{\#}$6, *Mela Sadvidhamargini*

70, IV$^{\#}$2^{b}7, *Mela Nasikabhusani,* Hungarian Major



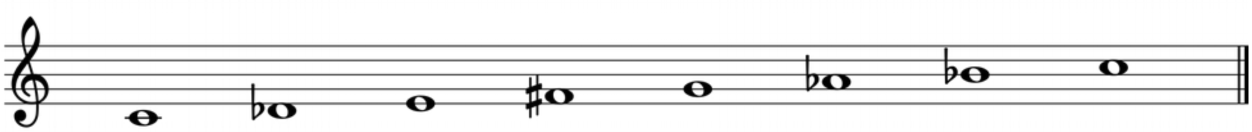
Singles, 3310

12, iii^{x}6$^{\#}$7, *Mela Rupavati*



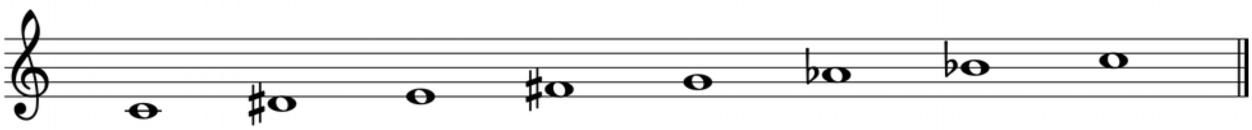
47, iii$^{\#}$4$^{\#}$6$^{\#}$7, *Mela Suvarnangi*

50, V^{b}2$^{\#}$4^{b}6, *Mela Namanarayani*



52, IV^{b}2^{b}7, *Mela Ramapriya,* Stravinsky's Petrouchka Chord

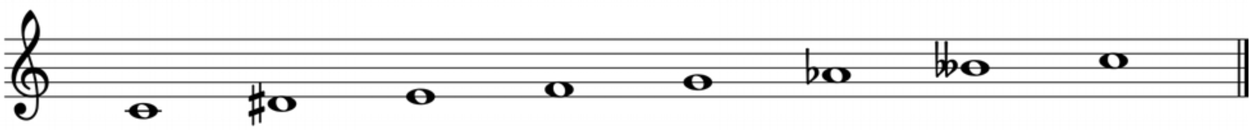
68, $\text{IV}^{\#}2^{b}6^{b}7$, *Mela Jyotisvarupini*



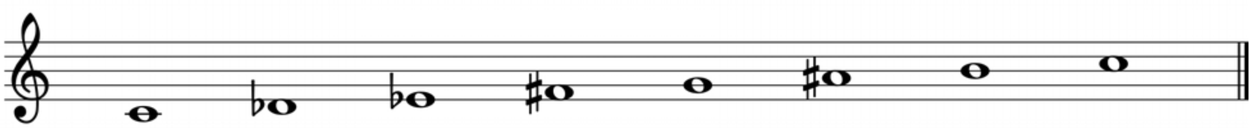
Singles 4120

6, $\text{iii}^{b}3^{x}6^{\#}7$, *Mela Tanarupi*

31, $\text{V}^{\#}2^{b}6^{b}7$, *Mela Yagapriya*

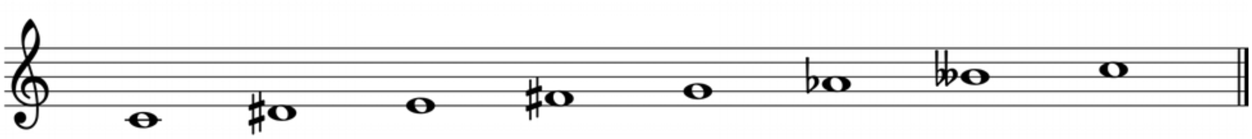


48, $\text{iii}^{\#}4^{x}6^{\#}7$, *Mela Divyamani*



49, $\text{IV}^{b}2^{b}6^{bb}7$, *Mela Dhavalambari,* Foulds' Mantra of Will Scale

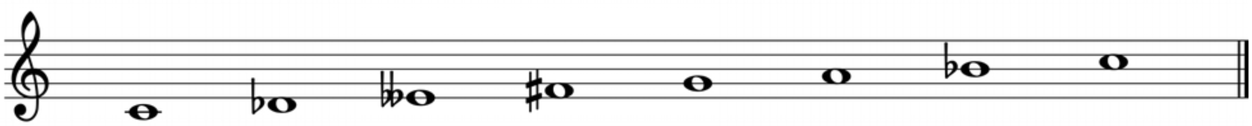
67, $\text{IV}^{\#}2^{b}6^{bb}7$, *Mela Sucharita*



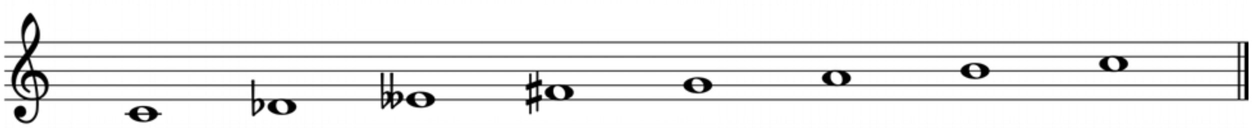
Singles 4201

38, $\text{iii}^{b}3^{\#}4$, *Mela Jhalarnavam*

40, $\text{iii}^{b}3^{\#}4^{\#}6$, *Mela Navanitam*



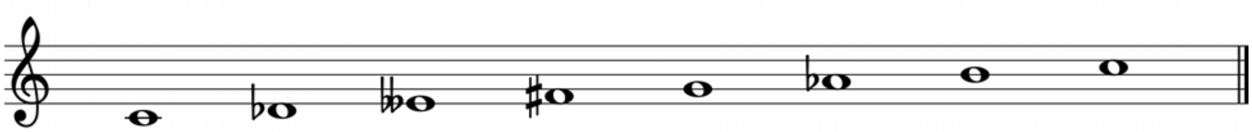
41, $\text{IV}^{b}2^{bb}3$, *Mela Pavani*



Singles 5011

37, $\text{iii}^{b}3^{\#}4^{b}7$, *Mela Salagam*

39, $\text{iii}^{b}3^{\#}4^{\#}7$, *Mela Jhalavarali*



42, $\text{IV}^{b}2^{bb}3^{\#}6$, *Mela Raghupriya*

The Periodic Table of Musical Scales

2500			3310							4210				4201	5011
8	10	11	9	14	2	16	4	24	12	3	1	33	6	38	37
65	64	62	66	71	53	27	25	32	47	54	51	60	31	40	39
28	26		56	21	19	59	5	46	50	55	13	36	48	41	42
20	23		35	58	7	30	61	70	52	15	69	45	49		
29					63	44			68	72	18		67		
22					17	34				57	43				

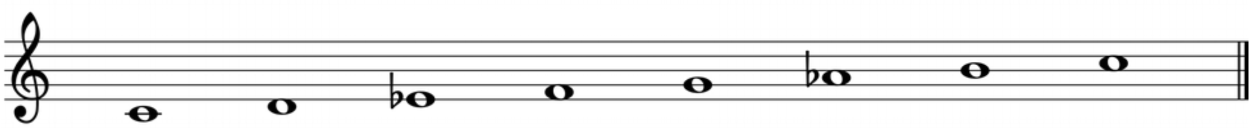
Chapter 4: The Periodic Table of Musical Scales

> Special knowledge can be a terrible disadvantage if it leads you too far along a path that you cannot explain anymore.
>
> Brian Herbert and Kevin J. Anderson
> *Dune: House Harkonnen*

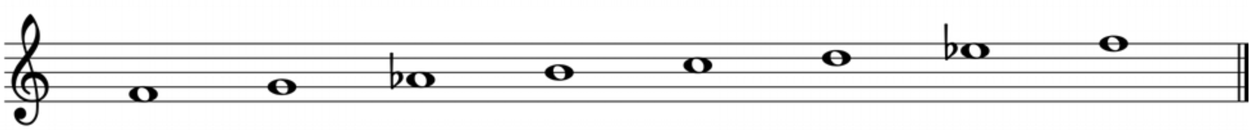
Displacement Groups

THE GROUPS OF SCALES formed by displacement have been described by the example of the six Western modes derived from the Major (and Minor) Scales. In South Indian Theory these sets of scales are known as *Grabdeha* Groups. They are most easily determined by summing intervals. We define the scales to be included as having a Tonic and Fifth, just like the original definition of *Melas*. We take each note in order and use it for the Tonic, then we test each scale for the presence of the Fifth. We find that the scales which pass this test are already members of the *Melakarta* system.

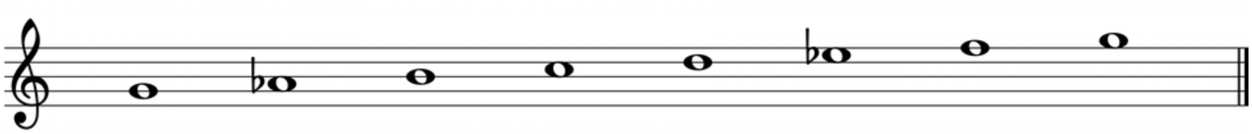
The Harmonic Minor Scale, Mode I



The Harmonic Minor Scale, Mode II



The Harmonic Minor Scale, Mode III

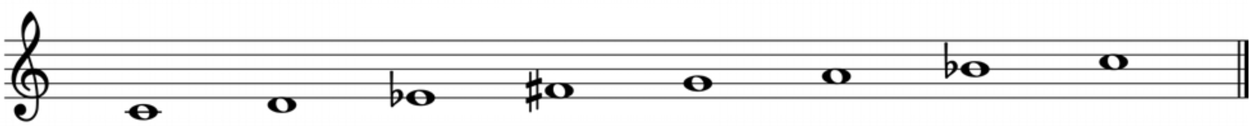


The Harmonic Minor Scale, Mode IV

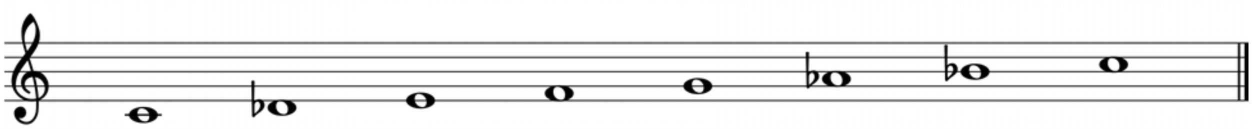
Having already introduced the six modes in Chapter 2, I will use the Harmonic Minor Scale as an example. Out of the seven possible Tonics, we find that only three contain a Fifth, making a total of four. As I have claimed, the four modes included in the Harmonic Minor system are also included in the *Melakarta* system. The above scales are put into the Tonic C.

21, vi$^{\sharp}$7, *Mela Kiravani*, *Pilu That*, Harmonic Minor

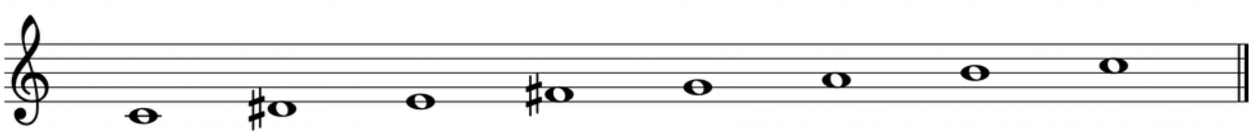
58, ii$^{\sharp}$4, *Mela Hemavati*



14, iii$^{\sharp}$3, *Mela Vakulabharanam*, Harmonic Major Inverse



71, IV$^{\sharp}$2, *Mela Kosalam*



Of course, these scales have no reference in Western Theory, except to say, "that's just the way they are." The Harmonic Minor has been used in classical music, where it is treated as an alteration of the Minor. Specifically, the Dominant Seventh is raised to the Major Seventh in order to provide a leading tone to the Tonic, and a major chord on the Fifth. I hope here to provide, as an extension of Western Theory, a rationale for the construction of this and other similar groups of scales.

So I will be using the Roman numeral system to represent the entire scale/mode/*mela* by adding, in order, the altered notes as flats or sharps. In this way the symbols for the chords can be extended to represent the entire scale. For instance ii is the Dorian Mode, and ii$^{\sharp}$7 changes the Seventh from Dominant to Major; this scale can be named the Harmonic Dorian. In the same way the Harmonic Minor can be represented as vi$^{\sharp}$7.

Interval Number Groups

Each *Mela* has a unique sequence of 1, 2, 3, or 4 intervals. In traditional theory these would be called Minor Second, Second, Minor Third, and Third. Because of the rule that all scales must have a Fifth, and all 7 scale intervals add up to 12, only these four small intervals can possibly occur, and only certain combinations as well. (The math is explained in the appendix.) The Interval Number is defined as the number of single intervals, the number of double intervals, the number of triple intervals, and the number of quadruple intervals written as a string of four numbers. So, for example, 2500 designates the scales that have 2 single intervals, 5 double intervals, 0 triple intervals, and 0 quadruple intervals. The Interval Numbers determine the "periods" of the Periodic Table of Musical Scales. For example, The Harmonic Minor Group has the Interval Number 3310, it has 3 single intervals, 3 double intervals, 1 triple interval, and 0 quadruple intervals. The 2500 Group has 12 scales, the 3310 Group (the largest) has 33 scales (coincidence), the 4120 Group has 21 scales, the 4201 Group has 3 scales, and the 5011 Group has 3 scales.

Melodic Motion Groups

The most useful aspect of the Interval Numbers is the first digit, that is, the number of single intervals in the scale. In each pair of adjacent notes, one or the other has the higher Consonance Value, therefore the Melodic Motion is determined by the movement of the lower value to the higher value. Each scale/mode/*mela* has a unique pattern. Taking examples from the Harmonic Minor Group, the Melodic Motions are:

Mela 21	2↑3:6↓5:7↑1
Mela 58	2↑3:4↑5:7↓6
Mela 14	2↓1:3↑4:6↓5
Mela 71	2↑3:4↑5:7↑1

This notation means that for *Mela* 21 the 2 resolves up to the 3, the 6 resolves down to the 5, and the 7 resolves up to the 1. Chapter 5 presents examples of how the Melodic Motion can be utilized for composition. Although this may seem a bit "touchy-feely," it can be said to determine a certain emotional response to the music.

Symmetrical Scales

Some of the scales also have internal symmetries of the mirror or glide types. On the Periodic Table, the mirror symmetrical scales are indicated by solid line boxes, and the glide symmetrical scales by dashed line boxes. Notice that 3 of the 10 symmetrical scales are members of the central group of 6. These scales, when compared to some of the assymetrical scales, may sound somewhat bland.

The mirror scales are:

Mela 6, iii^{b}3^{x}6$^{\#}$7
Mela 11, iii$^{\#}$6$^{\#}$7
Mela 15, I^{b}2^{b}6
Mela 22, ii
Mela 26, V^{b}6
Mela 31, V$^{\#}$2^{b}6^{b}7

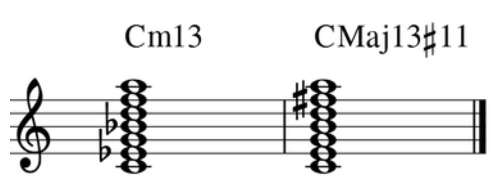
The glide symmetrical scales are:

Mela 1, iii^{b}3^{b}7
Mela 8, iii
Mela 29, I
Mela 36, I$^{\#}$2$^{\#}$6

Consonance Number Groups

I define consonance as the relative smoothness of notes sounding together. The numbers are based on the chapter "Helmholtz and Consonance" in *The Science of Musical Sound* by John R. Pierce. Each note of the total chromatic scale is placed on a "scale" of 1 to 12, with 12 (unison or octave) as the most consonant, and 1 (flat second) as the least.

The Consonance Values for each note of each scale are added together, without the Tonic and Fifth, because these are the same for every scale. This gives a range of values from 0 (least consonant) to 25 (most consonant). These totals are defined as the Consonance Numbers. When calculated in this manner, the Consonance Numbers of the *Melakartas* form a symmetrical array that resembles a normal curve, in other words there are few low numbers, a lot of middle numbers, and few high numbers. The six modes fall in the higher middle, except for the IV, which falls in the lower middle. These chords are an example.



The first chord is Dorian Mode, *Mela* 22, the ii of B^{b}, the Consonance Value is 21, high on a scale of 0 to 25. The second chord is Lydian Mode, *Mela* 65, the IV of G, the Consonance Value is only 10. These numbers provide a rational explanation for the fact that the Dorian Mode is much easier to harmonize than the Lydian Mode.

The entire concept of Consonance Values for each scale degree is admittedly controversial. Summing them up into a single Consonance Number for each scale is even more questionable.

The numbers I choose to use are extrapolated from the above source. Other determinations of Consonance Values exist. The details of these calculations are given in Appendix 1.

The Big 56 Scale Cheat Sheet

These are the 56 out of the 72 *melakartas* which fall into groups of at least 2. The remaining 16 singletons are not shown.

All you need to know is the six modes, the alterations are made to the 7 tone diatonic scale of the mode. For instance vi$^{\sharp}$7 is Aeolian mode with a sharp 7, that is the Harmonic Minor.

I recommend playing through them one group at a time, just like a beginner learning their scales on their instrument. For those of us trained in Western music, the scales become more exotic as we move down the chart; for those trained in Indian music, the *Bhairava*, *Shri*, and *Tori* groups may be more familiar.

Summary

The Periodic Table of Musical Scales describes the structure of each scale, as well as the relationships between the scales. The Interval Number Groups are the "periods" of the table. The first digit is the number of single intervals, the second digit is the number of double intervals, the third digit is the number of triple intervals, and the fourth digit is the number of quadruple intervals. The *Melakarta* Number denotes one of the 72 *Melakartas* (the South Indian system of scales). The Roman Numeral indicates the mode of the Western system including the sharp and flat alterations. The Intervals are simply the number of half steps from one note to the next. The Melodic Motion indicates the tendency of resolution from one note to another note. The Consonance Number is a measure of the relative smoothness of the harmonic structure imposed by the structure of the scale—a measure of the harmoniousness of the scale when considered as a chord; the value ranges from 0 for the most harsh to 25 for the most mellow. The mirror symmetrical scales are 6, 11, 15, 22, 26, and 31; the glide symmetrical scales are 1, 8, 29, and 36.

In the Periodic Table of Musical Scales the Six Universal Scales are highlighted in blue, the Three Groups of Four Scales in green, the Six Groups of Three Scales in yellow, the Ten Groups of Two Scales in red, the Ten Symmetrical Single Scales in purple, and the Six Asymmetrical Single Scales in gray. (I'm not sure if I can explain how these particular scales are symmetrical or not, except to say, "I know it when I see it.") There is no particular reason for this choice of colors; I just happened to use an editor's blue pencil to mark the Big Six, and then discovered the other groups.

Distribution of *Mela* Numbers by Consonance Numbers

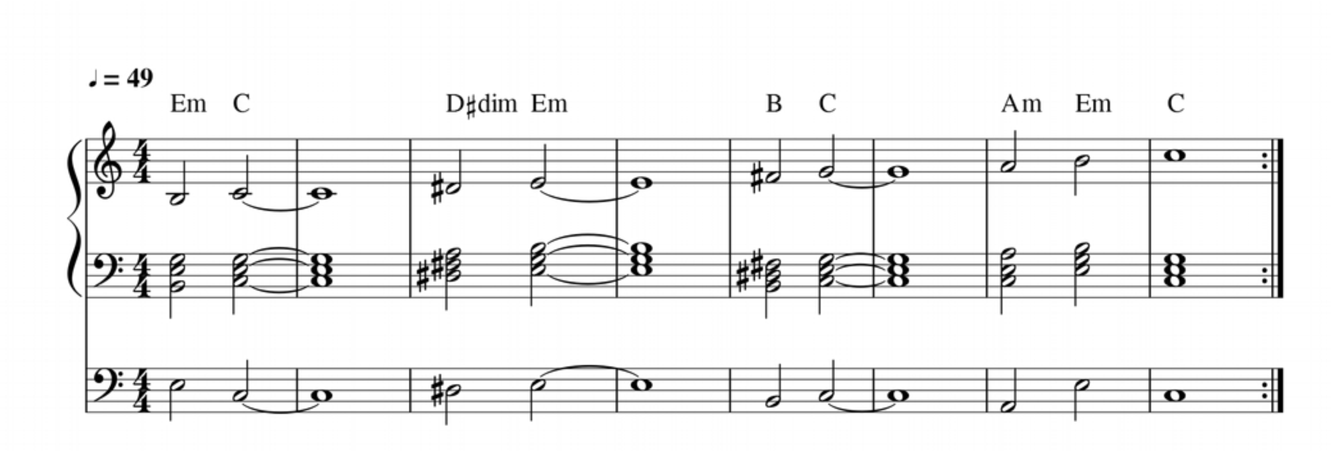
Consonance Number	*Mela* Numbers						Number of *Melas*
0	39						1
1							0
2	42						1
3							0
4	33	51					2
5	3	41	63				3
6	45	54					2
7	6	37	57	66			4
8	48	50					2
9	2	15	40	53	60	62	6
10	5	27	44	65			4
11	9	18	47	49	56	69	6
12	1	21	30	59	61		5
13	12	14	43	52	72		5
14	4	17	24	26	55	64	6
15	8	29	46	68			4
16	11	13	20	33	58	71	6
17	23	35					2
18	7	16	36	67			4
19	19	28					2
20	10	32	70				3
21	22	35					2
22							0
23	31						1
24							0
25	34						1

Chapter 5: Practical Applications for Improvisation and Composition

> The passion for science and the passion for music are driven by the same desire; to realize beauty in one's vision of the world.
>
> Heinz R. Pagels
>
> *Perfect Symmetry: The Search for the Beginning of Time*

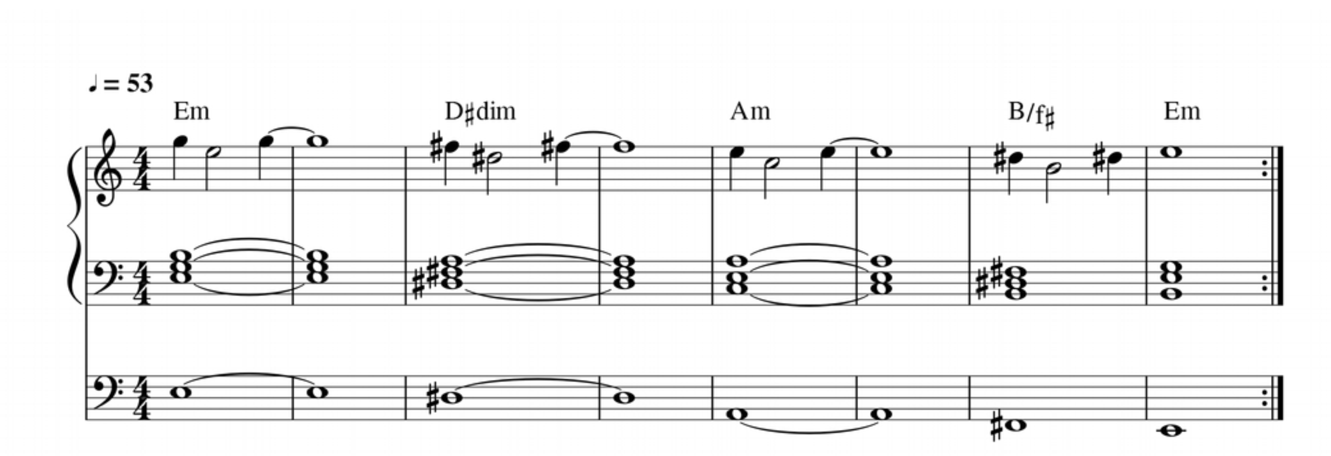
CONTINUING WITH THE EXAMPLES from the Harmonic Minor Group, here are four organ preludes. These little pieces are designed to make use of the Melodic Motion implied by the interval structures of each scale/mode.

Reverence



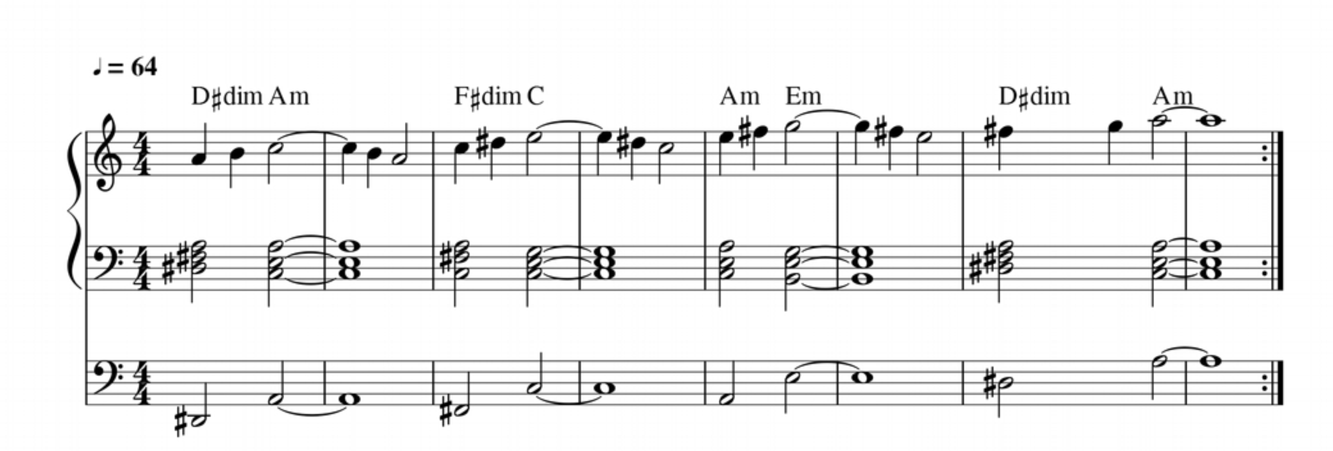
This piece is meant to be played at the beginning or ending of a service, when people are coming in or going out. The Tonic is C and the mode is *Mela* 71, or IV$^{\#}$2. Notice that the Interval Number (3310) tells us that there are three single intervals, and the Melodic Motion is all upwards. The somewhat ambiguous Em resolves up to the restful C, then the rather discordant D$^{\#}$diminished resolves up to a higher voicing of the Em, then the B resolves up to C, and finally the Am-Em sequence resolves up to the ultimate C.

Grief



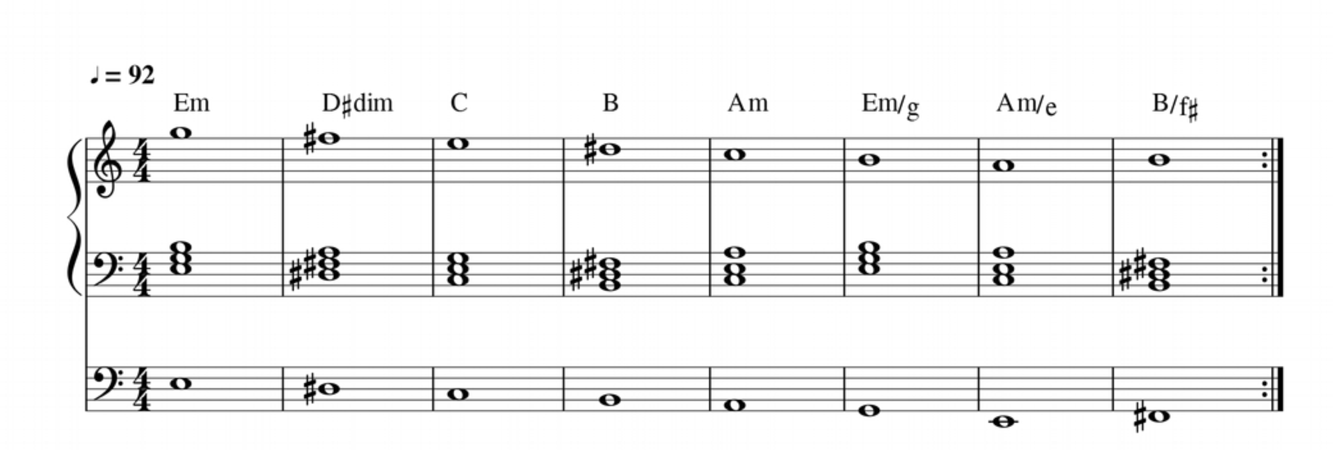
This is intended to be played at a funeral or memorial. The melody line moves very little, down and up, down and up again, and finally back up to Em. The scale is the Harmonic Minor on Tonic E, *Mela* 21, vi$^{\#}$7.

Pride



This is to be played at confirmations and graduations. The scale is *Mela* 58, ii$^{\#}$4, on the Tonic A. The melody moves as three instances of gain and loss, ending with a triumphant gain.

Love



This is meant to be played at weddings. The scale is *Mela* 14, iii$^{\#}$3 on the Tonic B. The descending melody line represents falling in love.

Although the music notation software forces the use of all accidentals, as in Twelve Tone scores, the group should be thought of as a single key signature. The two sharps are always present. There is plenty of room for improvisation; a first step might be to arpeggiate the left hand chords.

Now I will turn from the majestic pipe organ to the lowly mouth organ, more commonly known as the harmonica or simply the harp. Lee Oskar has advanced the art of the harmonica by providing additional tunings beyond the standard Major: Natural (Related) Minor, Harmonic Minor, and one he calls Melody Maker. The brochure says, "Our altered tunings make it easier to play many different styles and explore new musical directions." Believe it.

The harp, in the standard Hohner Marine Band form, is a diatonic instrument—each instrument is built in a single key. The full set of 12 keys ranges from Violin G up to Middle F$^{\#}$. Most harp players play the Chicago blues style, which is also known as cross harp; for instance in a blues in the key of G, the harp player will use the C harp. Also, many harp players are non-reading, so they use positions rather than staff notation. (Please don't be offended by this. I need the written music for the pipe organ, but the harp is intuitive for me.) The Lee Oskar brochure has handy charts for selecting the right harp for each key (and mode). Here is a simplified version using only Tonic C:

Harp Key Position	1st	2nd	3rd	4th
Major C	Major C	Dominant G	Dorian Dm	Minor Am
Melody Maker C	Dorian Dm	Major C	Dominant G	Minor Am
Natural Minor Cm	Dorian Fm	Minor Cm	Major E^{b}	Dominant B^{b}
Harmonic Minor Cm	Minor Cm	Major E^{b}	Dominant B^{b}	Dorian Fm

Here is my revised and expanded chart using the Roman Numeral notation:

Harp Key Position	1st	2nd	3rd	4th	5th	6th
Major	I	V	ii	vi	iii	IV
Melody Maker	ii	I	V	vi	IV	iii
Natural Minor	ii	vi	I	V	iii	IV
Harmonic Minor	vi$^{\#}$7	iii$^{\#}$3	ii$^{\#}$4	IV$^{\#}$2	-	-

Notice that, while there are 6 modes in the Major/Minor scales, there are only 4 in the Harmonic Minor. The Melody Maker tuning is, surprisingly, good for the IV or Lydian Mode.

The Big 56 Scale Cheat Sheet

Scale Group						
Universal	I	ii	iii	IV	V	vi
Melodic Minor		ii$^{\#}$7	iii$^{\#}$6	IV^{b}7	V^{b}6	
Neapolitan Minor	I$^{\#}$2		iii$^{\#}$7	IV$^{\#}$6		vi$^{\#}$4
Harmonic Minor		ii$^{\#}$4	iii$^{\#}$3	IV$^{\#}$2		vi$^{\#}$7
Marva			iii^{b}3	IV^{b}2		vi^{b}7
Syam			iii^{b}3$^{\#}$7	IV^{b}2^{b}6		vi$^{\#}$4^{b}7
Bhairubahar	I^{b}2		iii^{b}7	IV^{b}6		
Bhairava	I^{b}2^{b}6			IV$^{\#}$2$^{\#}$6		vi$^{\#}$4$^{\#}$7
Harmonic Major	I^{b}6			IV^{b}3	V^{b}2	
Naga	I$^{\#}$6		iii$^{\#}$4		V$^{\#}$2	
Shri			iii^{b}3^{b}7	IV^{b}2^{b}6		
Vanaspati			iii^{b}3$^{\#}$6			vi$^{\#}$3^{b}7
Manavati	I^{b}2^{bb}3			IV^{b}6^{bb}7		
Whole Tone			iii$^{\#}$6$^{\#}$7	IV^{b}6^{b}7		
Gayaka			iii$^{\#}$3^{b}7	IV$^{\#}$2^{b}6		
Kambari	I^{b}2$^{\#}$6		iii$^{\#}$4^{b}7			
Varuna		ii$^{\#}$6$^{\#}$7				vi$^{\#}$2$^{\#}$3
Ganga	I$^{\#}$2^{b}6			IV^{b}3$^{\#}$6		
Todi	I$^{\#}$2$^{\#}$6		iii$^{\#}$4$^{\#}$7			
Hungarian Major			iii$^{\#}$4$^{\#}$6	IV$^{\#}$2^{b}7		

Conclusion

> Discrete symmetries have minimum nonzero steps—hence "discrete" symmetry operations. A continuous symmetry, with its infinite number of symmetry operations, is a "bigger" symmetry than a discrete symmetry. Hence, continuous symmetries are very powerful constraints on the structure of space and time. It turns out that it is actually mathematically easier to analyze continuous symmetries, because the powerful techniques of differential calculus can be brought to bear, whereas discrete symmetries pose many challenging counting problems in their analysis.
>
> Leon M. Lederman and Christopher T. Hill
> *Symmetry and the Beautiful Universe*

AND SO, GENTLE READER, you have been promised a quantum theory of music...

Such a theory must be composed of analogy and metaphor. It can only describe a small part of the existing raw materials of music—that is the scales and modes. The "quantum" is the note itself. The heptatonic/dodecatonic system is recognized as one of many possible systems. The theory selects and classifies the most useful modes, analogous to the quantum real world, where the combinations of physical particles are the raw material for the chemical elements; the combinations of notes build the modes, and define their positions on the "periodic table." Continuing the analogy, the modes range from left to right, from common to rare, from harmonious to dissonant. The positions of the 72 *melakartas* can be said to resemble the 92 natural chemical elements. The 120 modes form what I am pleased to call the *Ekamelakarta* system, which is (trumpet fanfare) isomorphic with the 120 symmetries of the icosahedron, as described by Felix Klein. Technically the *Ekamelakartas* would be the 48 *melas* beyond the 72 "natural" *melakartas*.

My first glimpse of the geometric symmetries of musical form came as the (at the time) arbitrary assignment of the 12 keys to the faces of the dodecahedron or the vertices of the icosahedron. Either polyhedron can be "stellated" by adding pyramids to the sides, pentagonal on the dodecahedron and triangular on the icosahedron, giving the totals: $12 \cdot 5 = 60$ and $20 \cdot 3 = 60$, respectively. We can take these to be 12 keys times 5 chords giving 60 chords, as I learned a half century ago. If you learn all 12 Major Scales and 5 chord forms (Major Seventh, Dominant Seventh, Sixth, Minor Seventh, and Half Diminished), then you can reach a high level of skill on your instrument. Practice, practice, practice!

The modes naturally form themselves into groups, in both the common meaning of the word "group" and the more precise mathematical term. The *Grabdeha* Groups are glide groups formed by a rule: glide up to the next scale degree, if the new mode has a perfect fifth, then it is a member of the group. Some of the modes are also mirror and glide symmetrical; the positions of these modes are obvious on the *Melakarta* Table, but somewhat surprising on the Periodic Table. The classifications of the modes into various groups can provide an expanded understanding of their characteristics for the practical purposes of composition and improvisation.

The math here is descriptive and intuitive, focusing on practical use. There are no theorems and no rigorous proofs. The numbers are all finite integers; the equations are all simple algebra. Yes, we need trigonometry and calculus to analyze musical waveforms, but here we are only concerned with the named notes produced by actual instruments. So, rather than counting the notes from 0 to 11, as a mathematician would do, I count from 1 to 12, as a musician does. We are, after all, talking about positions, not quantities; ordinal, not cardinal numbers.

Regarding theories, there are two kinds, those which are practically constructed from real world data, and those which are generated from mathematical induction. Considering the current state of physics, we have the Standard Model, which works in the real world, but obstinately refuses to combine with General Relativity. The group called SU(5) has failed to provide the structure for a Grand Unification, but this group is also the 120 member group of the *Ekamelakartas*. Music Theory only describes human constructs, not the fundamental laws of the universe.

Quantum physics in the form of the Standard Model has produced our electronic revolution. It works in the real world. Two other theories have also played their parts; Sound Theory (Acoustics) and Color Theory. These two last mentioned are descriptive of human perceptions; they would be different if the human senses of hearing and sight were different. The electronics of sound systems and video screens are designed to fit our human senses. So we see that the greatest advances in science are now perverted into commercial advertising and political propagandizing. Meanwhile, the promise of unlimited power from nuclear fusion is indefinitely postponed.

My greatest fear is not a zombie apocalypse, it is technology without science. This is the very meaning of Cordwainer Smith's Instrumentality, the physical constructions and the craft expertise to build and maintain an empire. Technology without science is oppression. But what is science without technology? Philosophy? Mathematics? Ethics?

Back to Music Theory: We take a set of 12 tones and build a subset of 7, then we make certain specifications regarding the separation into the tetrachords and the requirement for the perfect fifth. Simple combinatorics allow us to enumerate the modes within these defined sets. Simple algebra allows us to place them into groups. The 7 of 12 or diatonic system is only one of many possible ways to divide the octave. It just so happens that the human ear divides tones in approximately equal intervals. That is, everybody has the same basic range of hearing from low tones to high tones, and everybody has the same discrimination between closely adjacent tones. A 12 tone division of the octave is a very good model of human hearing, and therefore has become almost universal throughout the world. The *Melakarta* system is the best I have found for constructing and classifying scales.

Appendix 1 describes the relations among the scales of this system and how the periodic table is built and interpreted. It also expains how the Consonance Values are calculated, as well as the symmetrical scales.

Appendix 2 describes a simplified 5 of 8 system, just for the theoretical interest of such an exercise, maybe I'll have the time and money to build actual octatonic instruments...or maybe somebody else will take up an interest in realizing such instruments.

Appendix 1: The Mathematics of the Musical Scales

MANY DIFFERENT METHODS of tuning have been used throughout the ages and even more have been proposed during modern times. For my purposes here, we will use the twelve tone equal temperament tuning. The set of the 12 notes of the chromatic scale form a ring mod 12, which is theoretically infinite. However, in practical use the human hearing range of frequencies determines the lower and upper limits. The piano keyboard has a well known range of 88 notes, and the MIDI (Musical Instrument Digital Interface) system uses a range of 128 notes.

$S_T = \{f_{min}, \ldots, f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}, f_{11}, f_{12}, \ldots, f_{max}\}$
$= \{f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}, f_{11}, f_{12}\}$
$= \{1, 2 , 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
$= \{0, 1, 2 , 3, 4, 5, 6, 7, 8, 9, 10, 11\}$
$= \{C, C^{\#}/D^{b}, D, D^{\#}/E^{b}, E, F, F^{\#}, G, G^{\#}/A^{b}, A, A^{\#}/B^{b}, B\}$

For simplicity I will begin by using the clock face form:

$S_T = \{1, 2 , 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

In the *Melakarta* system the modes or scales are built from 7 of 12 notes divided into two sets (tetrachords). The {1} and the {8} are required, so the *Melakarta* is defined by:

M = {1} + 2 of {2, 3, 4, 5} + 1 of {6, 7} + {8} + 2 of {9, 10, 11, 12}

This gives a total number of combinations: 4C2 · 2C1 · 4C2 = 6 · 2 · 6 = 72

Another, larger set can be defined by:

M' = {1} + 3 of {2, 3, 4, 5, 6, 7} + {8} + 2 of {9, 10, 11, 12}

Which gives the combinations: 6C3 · 4C2 = 20 · 6 = 120

So we see that the *Melakarta* system is a subset of a larger set. But we already had some idea that it should be so. Any 7 of 12 is another possible set of notes:

12C7 = 792, which large number is practically useless for musical purposes.

We can also define a smaller subset of the *Melakarta* system by:

M'' = {1} + 1 of {2, 3} + 1 of {4, 5} + 1 of {6, 7} + {8} + 1 of {9, 10} + 1 of {11, 12}

Which gives the combinations:

$2C1 \cdot 2C1 \cdot 2C1 \cdot 2C1 \cdot 2C1 = 2^5 = 32$

This 32 mode system is very nice for study. It forms the inner 4 by 4 squares of the two 6 by 6 squares that form the *Melakarta* system.
We can also build the *Melas* from intervals:

$I = \{i_1, i_2, i_3, i_4, i_5, i_6, i_7\}$

$i_1 = 1$ of $\{1, 2, 3\}$
$i_2 = 1$ of $\{1, 2, 3\}$
$i_3 = 1$ of $\{1, 2, 3, 4\}$
$i_4 = 1$ of $\{1, 2\}$
$i_5 = 1$ of $\{1, 2, 3\}$
$i_6 = 1$ of $\{1, 2, 3\}$
$i_7 = 1$ of $\{1, 2, 3\}$

$I_{total} = i_1 + i_2 + i_3 + i_4 + i_5 + i_6 + i_7 = 12$
$I_{lower} = i_1 + i_2 + i_3 + i_4 = 7$
$I_{upper} = i_5 + i_6 + i_7 = 5$

I_{lower} has 12 sets of solutions:

i_1	i_2	i_3	i_4
1	1	4	1
1	2	3	1
1	3	2	1
2	1	3	1
2	2	2	1
3	1	2	1
1	1	3	2
1	2	2	2
1	3	1	2
2	1	2	2
2	2	1	2
3	1	1	2

And I_{upper} has 6 sets of solutions:

i_5	i_6	i_7
1	1	3
1	2	2
1	3	1
2	1	2
2	2	1
3	1	1

So the total number of solutions is then $12 \cdot 6 = 72$, which is simply another way to describe the *Melakartas*.

The number of times each interval occurs in each scale can also be calculated. The subsets defined by these numbers can be very useful in classifying the scales.

$X = \{x_1, x_2, x_3, x_4\}$

$x_n = 1$ of $\{0, 1, 2, 3, 4, 5\}$

$$x_1 + 2x_2 + 3x_3 + 4x_4 = 12$$
$$x_1 + x_2 + x_3 + x_4 = 7$$
$$x_2 + 2x_3 + 3x_4 = 5$$

Which has 5 solution sets:

x_1	x_2	x_3	x_4
2	5	0	0
3	3	1	0
4	1	2	0
4	2	0	1
5	0	1	1

Each of these solution sets describes a subset of the scales we have considered, the *Melakarta* system (72 scales), and the larger 120 scale system, which I will call the *Ekamelakarta* system. The *Ekamelakartas* are all of the possible combinations when the scales are built from tetrachords requiring the perfect fifth. Of course, the tetrachords provide the key to our puzzle. The 12 tones are divided into the two sets of 7 and 5. So we find the 4 membered partitions of 7 and the 3 membered partitions of 5.

4 membered partitions of 7	Permutations	
	Ekamelakartas	*Melakartas*
4 1 1 1	4	1
3 2 1 1	12	7
2 2 2 1	4	4
Sum	20	12

3 membered partitions of 5	Permutations	
	Ekamelakartas	*Melakartas*
3 1 1	3	3
2 2 1	3	3
Sum	6	6

So we see that the *Ekamelakartas* number $20 \cdot 6 = 120$, and the *Melakartas* number $12 \cdot 6 = 72$. These results are the same as we have calculated earlier by different methods.

Now let's return to the set $X = \{x_1, x_2, x_3, x_4\}$, and the practical application of these solutions. In plain language the solution set {2, 5, 0, 0} means that the scales in this set have 2 single intervals, 5 double intervals, no triple intervals, and no quadruple intervals. The next solution set {3, 3, 1, 0} gives 3 singles, 3 doubles, 1 triple, and no quadruple intervals, and so forth. So these sets form the "periods" of the modes.

Table of Consonance Values

Diatonic Scale Degree	Number	Name	Consonance Value
Tonic	1	C	12
Minor Second	2	D^b	1
Major Second	3	D/E^{bb}	2
Minor Third	4	$D^\#/E^b$	8
Major Third	5	E	6
Fourth	6	F	9
Sharp Fourth	7	$F^\#$	4
Fifth	8	G	11
Minor Sixth	9	A^b	5
Major Sixth	10	A/B^{bb}	10
Dominant Seventh	11	$A^\#/B^b$	7
Major Seventh	12	B	3

The Consonance Numbers are calculated by enumerating each scale degree except the 1 (value 12) and 8 (value 11) which are not used because these are present in every scale. Using the integers 1 through 10, we choose 5 numbers and add them.

Maximum total of 5 of 10 = 6 + 7 + 8 + 9 + 10 = 40
Minimum total of 5 of 10 = 1 + 2 + 3 + 4 + 5 = 15

40 – 15 = 25,

or, for the real mathematicians:

$$\sum_{x=6}^{10} x - \sum_{x=1}^{5} x = 25$$

Which gives a range of values from 0 to 25. Here are two sample calculations, one for Dorian Mode and another for Lydian Mode:

Mela 22 = {1, 3, 4, 6, 8, 10, 11}
C values = {2, 8, 9, 10, 7}
Consonance = 2 + 8 + 9 + 10 + 7 = 35 – 15 = 21

Mela 65 = {1, 3, 5, 7, 8, 10, 12}
C values = {2, 6, 4, 10, 3}
Consonance = 2 + 6 + 4 + 10 + 3 = 25 – 15 = 10

Symmetrical Scales

The scale symmetries are determined by matching the interval patterns of the lower and upper tetrachords. Here are two examples:

Mela 6 = {1 1 3 2 3 1 1}, mirror symmetry, and
Mela 1 = {1 1 3 2 1 1 3}, glide symmetry.

Notice that these symmetries only occur when the fourth interval is 2, and that they fall on the diagonals of the first 36 *Melakartas*.

Appendix 2: A Theoretical 5 of 8 System

NOW I WANT TO PRESENT a wholly speculative system of scales built of five notes out of an eight note tempered chromatic. For the first time in this booklet we must actually calculate the frequencies of the notes.

$$f_1 = 261.6 \text{ Hz}, \; f_n = f_{n-1} \cdot \sqrt[8]{2}, \; (2 \leq n \leq 9)$$

Note	1	2	3	4	5	6	7	8
Freq. Hz	261.6	285.3	311.1	339.3	370	403.4	440	479.8

To build the system similar to the *Melakartas*, the scale is divided into lower and upper trichords (analogous to the tetrachords). We require both the 1 and the 6, which results in:

S = {1} + 2 of {2, 3, 4, 5} + {6} + 1 of {7, 8}
4C2 · 2C1 = 6 · 2 = 12 scales

	12	21
113	1	2
122	3	4
131	5	6
212	7	8
221	9	10
311	11	12

S_1 = { 1 1 3 1 2 }
S_2 = { 1 1 3 2 1 }
S_3 = { 1 2 2 1 2 }
S_4 = { 1 2 2 2 1 }
S_5 = { 1 3 1 1 2 }
S_6 = { 1 3 1 2 1 }
S_7 = { 2 1 2 1 2 }
S_8 = { 2 1 2 2 1 }
S_9 = { 2 2 1 1 2 }
S_{10} = { 2 2 1 2 1 }
S_{11} = { 3 1 1 1 2 }
S_{12} = { 3 1 1 2 1 }

Group 1 = { S_3 S_7 S_8 S_{10} }
Group 2 = { S_4 S_9 }
Group 3 = { S_1 S_6 }
Group 4 = { S_5 S_{12} }
Group 5 = { S_2 }
Group 6 = { S_{11} }

S_4 and S_7 have mirror symmetry, and S_3 and S_8 have glide symmetry.
Here is the periodic table, with the Interval Group Numbers:

230		311			
3	4	1	5	2	11
7	9	6	12		
8					
10					

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